

RE-EXAMINATION OF THE PM-METHOD

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Abstract—The application of the PM-method¹ has been tested and some cases where it does not apply are discussed.

ACCORDING to the quantum-mechanical theory of dispersion,² the molecular rotation, $[M]_{\lambda}$ is given by:

$$[M]_{\lambda} = \{288 \pi^2 N / \lambda^2\} \{(n_{\lambda}^2 + 2)/3\} g_{\lambda} \quad (1)$$

Here, the various physical constants have their usual meaning. Kirkwood concluded³ that the total gyration parameter, g_{λ} is given by:

$$g_{\lambda} = g_{\lambda}^{(0)} + g_{\lambda}^{(1)} + \sum_i g_{\lambda}^{(i)} \quad (2)$$

where⁴ the first term $g_{\lambda}^{(0)}$ represents the dynamic coupling effect which originates from the electronic correlative interactions between different groups. The second term $g_{\lambda}^{(1)}$ has not been studied in detail and is usually neglected; $g_{\lambda}^{(i)}$ the third term represents the one-electron effect, i.e., the intrinsic rotation of each group due to the motion of an electron in the asymmetric Hartree field.

By assuming

$$g_{\lambda} \doteq g_{\lambda}^{(0)} \quad (3)$$

and combining Eq. 3 with Eq. 1, Kirkwood proposed:

$$[M]_{\lambda} = \{288 \pi^2 N / \lambda^2\} \{(n_{\lambda}^2 + 2)/3\} g_{\lambda}^{(0)}. \quad (4)$$

In the previous paper,¹ this Eq. 4 for the wave-length of Na D-line was applied with some modification of symbols as

$$[\mu]_{\text{D calcd}(\text{XY})}^{20} = \{288 \pi^2 N / \lambda^2\} \{(n^2 + 2)/3\} g_{\text{XY}}. \quad (5)$$

As the neglect of two terms,

$$g^{(1)} \quad \text{and} \quad \sum_i g^{(i)},$$

as well as other factors may cause a discrepancy between the computed and the observed optical rotations, a new adjustable parameter κ has been introduced as in:

$$\kappa_{\text{XY}} [\mu]_{\text{D calcd}(\text{XY})}^{20} = [\mu]_{\text{D obs}(\text{XY})}^{20} \quad (6)^5$$

¹ S. Yamana, *J. Amer. Chem. Soc.* **86**, 1606 (1964).

² J. G. Kirkwood, *J. Chem. Phys.* **5**, 479 (1937).

³ J. G. Kirkwood, *J. Chem. Phys.* **7**, 139 (1939).

⁴ H. Murakami, *J. Chem. Phys.* **27**, 1231 (1957).

⁵ S. Yamana, *Bull. Chem. Soc. Japan* **31**, 558 (1958).

and further, for simplification, κ was factorized as

$$\kappa_{XY} = \zeta_X \zeta_Y. \quad (7)^5$$

Thus, from Eqs. 6 and 7,

$$\zeta_X \zeta_Y [\mu]_{D \text{ calcd}(XY)}^{20} = [\mu]_{D \text{ obs}(XY)}^{20} \quad (8)^1$$

Equation 8 is the fundamental formula in the PM-method which relates the Kirkwood's term, $[\mu]_{D \text{ calcd}(XY)}^{20}$ to the partial molecular rotation, $[\mu]_{D \text{ obs}(XY)}^{20}$ and it has been shown that Eq. 8 is applicable in a number of cases.¹ It is probable, however, that in spite of introduction of κ - or ζ -coefficients the result of neglecting $g_\lambda^{(1)} + \sum_i g_\lambda^{(i)}$ cannot be overcome completely and, in some cases, causes a discrepancy between the computed and the observed optical rotations. Consequently, the PM-method has been re-examined (especially Eqs. 6 and 7). The compounds and their observed molecular rotations are listed in Tables 1, 2 and 3.

TABLE 1

Cyclohexane derivative	Unit group	$[M]_D^{20}$ °	Ref.
(-)- <i>trans</i> -1,2-Cyclohexanediol	[(OH) ^{1β} , (OH) ^{2α}]	-54.0°	7
(-)- <i>trans</i> -2-Aminocyclohexanol	[(OH) ^{1β} , (NH ₂) ^{2α}]	-46.3°	8
(-)- <i>trans</i> -2-Chlorocyclohexanol	[(OH) ^{1β} , Cl ^{2α}]	ca. -53.8°	9
(-)- <i>trans</i> -2-Methylcyclohexanol	[(OH) ^{1β} , (CH ₃) ^{2α}]	-43.9°	10
(-)- <i>trans</i> -2-Chloro-1-aminocyclohexane	[(NH ₂) ^{1β} , Cl ^{2α}]	-59.3° ¹¹	12
(-)- <i>trans</i> -2-Methyl-1-aminocyclohexane	[(NH ₂) ^{1β} , (CH ₃) ^{2α}]	-29.1° _{a1} ¹¹	12
(+)- <i>cis</i> -3-Methylcyclohexanol	[(OH) ^{1β} , (CH ₃) ^{2β}]	2.2°	13
(+)-1,3-Dimethylcyclohexane	[(CH ₃) ^{1β} , (CH ₃) ^{2α}] ¹⁴	1.4°	15

al: alcoholic solution.

TABLE 2

Cyclopentane derivative	Unit group	$[M]_D^{20}$ °	Ref.
(-)- <i>trans</i> -1,2-Cyclopentanediol	[(OH) ^{1β} , (OH) ^{2α}]	-34.3° _w ¹¹	16
(+)- <i>trans</i> -2-Aminocyclopentanol	[(OH) ^{1α} , (NH ₂) ^{2β}]	38.6°	17
(-)- <i>trans</i> -3-Methylcyclopentanol	[(OH) ^{1β} , (CH ₃) ^{2α}]	-6.5°	18
(+)- <i>trans</i> -1,3-Dimethylcyclopentane	[(CH ₃) ^{1β} , (CH ₃) ^{2α}]	2.3°	19

w: water solution.

⁶ Some of $[M]_D^{20}$ values were assumed from $[M]_D$ which had been observed in the neighbourhood of 20°.

⁷ N. A. B. Wilson and J. Read, *J. Chem. Soc.* 1269 (1935).

⁸ R. M. Godchot and M. Mousseron, *C.R. Acad. Sci., Paris* 194, 981 (1932).

⁹ F. J. Zeelen, M. E. Kronenberg and E. Havinga, *Rec. Trav. Chim.* 77, 674 (1958).

¹⁰ G. A. C. Gough, H. Hunter and J. Kenyon, *J. Chem. Soc.* 2052 (1926).

¹¹ For lack of data, $[\alpha]_D$ is computed by applying the data of $[\alpha]_{546}$ and $[\alpha]_{576}$ to Drude's single term Eq., $[\alpha]_\lambda = A/(\lambda^2 - \lambda_0^2)$

¹² M. Mousseron and P. Froger, *Bull. Soc. Chim. Fr.* 14, 843 (1947).

¹³ D. S. Noyce and D. B. Denny, *J. Amer. Chem. Soc.* 74, 5912 (1952).

¹⁴ The author expresses his great thanks to Prof. James H. Brewster (Purdue University) for his experiments to re-confirm the absolute configuration of (+)-*trans*-1,3-dimethylcyclohexane.

¹⁵ M. Mousseron, *Bull. Soc. Chim. Fr.* 218 (1946).

¹⁶ M. Mousseron and R. Richaud, *Bull. Soc. Chim. Fr.* 643 (1946).

¹⁷ M. Mousseron and R. Granger, *Bull. Soc. Chim. Fr.* 850 (1947).

¹⁸ M. Godchot, G. Cauquil and M. R. Calas, *Bull. Soc. Chim. Fr.* 6, 1358 (1939).

¹⁹ S. F. Birch and R. A. Dean, *J. Chem. Soc.* 2477 (1953).

TABLE 3

Menthol-like compound	Unit group	$[M]_D$	Ref.
(+)-Neomenthylamine	$[(CH_3)^{1\beta}, (NH_2)^{2\alpha}, (iso-C_3H_7)^{4\alpha}, ch.R]^{20}$	23.5°	21

In order to simplify computations, molecular models are assumed as follows; The cyclohexane-ring exists only in C1 conformation (i.e. the number of equatorial groups becomes as many as possible in Tables 1, 2 and 3) and the cyclopentane-ring plane is a flat regular pentagon and the length of its side is 1.54 Å. The angle, $109^\circ 28'$, between the two bonds which project up and down the cyclopentane-ring from the same ring carbon atom is bisected by the cyclopentane-ring plane.²³ The characteristics used in this article are as below:

Bond length: C—C bond is 1.54 Å; C—O bond is 1.42 Å; C—N bond is 1.47 Å; C—Cl bond is 1.78 Å. Mean polarizability, α^{25} : α of OH group is 1.04×10^{-24} cc; α of NH₂ group is 1.79×10^{-24} cc; α of Cl atom is 2.36×10^{-24} cc; α of CH₃ group is 2.27×10^{-24} cc. Anisotropy ratio, β^{26} : β of OH group is 0.35; β of NH₂ group is 0.10; β of Cl atom is 0.33; β of CH₃ group is 0.35. Refractive index, n : n of cyclohexane derivative is n_1 ; n of cyclopentane derivative is n_2 ; n of carbohydrate is n_3 ; n of menthol-like compound is n_4 .

The $[\mu]_D^{20} \text{ calcd}(XY)\{3/(n^2 + 2)\}$ values are calculated by Eq. 5.¹ These computed values are given in Tables 4, 5 and 6.

TABLE 4. $[\mu]_D^{20} \text{ calcd}(XY)\{3/(n^2 + 2)\}$, (= (X) × (Y)), CAUSED BY THE DYNAMIC COUPLING EFFECT BETWEEN TWO UNIT GROUPS, X AND Y, IN CYCLOHEXANE DERIVATIVE

		C1 Conformation					
		Y ^{4β}	Y ^{4α}	Y ^{3β}	Y ^{3α}	Y ^{2β}	Y ^{2α}
X ^{1α}		0	0	G	0	0	E
X ^{1β}		0	0	0	-F	E	-E
X	Y	E ²⁷		F ²⁷		G ²⁷	
OH	OH	11.73 (6.50)		1.88 (1.66)		1.88 (1.66)	
OH	NH ₂	5.75 (3.03)		0.93 (0.79)		0.92 (0.80)	
OH	Cl	17.50 (11.29)		5.38 (3.70)		2.87 (2.79)	
OH	CH ₃	25.30 (13.35)		4.20 (3.70)		4.04 (3.34)	
NH ₂	Cl	8.60 (5.48)		2.63 (1.76)		1.42 (1.38)	
NH ₂	CH ₃	12.39 (6.43)		2.06 (1.76)		2.00 (1.66)	
Cl	CH ₃	38.15 (23.81)		6.38 (6.22)		11.53 (7.44)	
CH ₃	CH ₃	54.54 (27.64)		9.02 (7.45)		9.02 (7.45)	

²⁰ Owing to the steric repulsion of (NH₂)^{2α}, the symmetrical type is only one possible type of the internal conformation of the isopropyl group. In this case, the following simplification may be introduced,

$$(iso-C_3H_7)^{4\alpha} \cong (CH_3)^{4\alpha} + (CH_3)^{\alpha\beta-d} + (CH_3)^{\alpha\beta-e} \quad (\text{cf. } 22).$$

²¹ J. Read and R. G. Johnston, *J. Chem. Soc.* 226 (1934).

²² S. Yamana, *Bull. Chem. Soc. Japan* 34, 1414 (1961).

²³ Strictly speaking, the cyclopentane-ring is not a perfectly flat plane (ref. 24).

²⁴ K. S. Pitzer and W. E. Donath, *J. Amer. Chem. Soc.* 81, 3213 (1959).

²⁵ Landolt Börnstein, *Physikalisch-Chemische Tabellen* Hw. II, p. 985.

²⁶ Landolt Börnstein, *Physikalisch-Chemische Tabellen* 5th Ed., Eg. II, p. 90, 91; Eg. III, p. 1205.

²⁷ Given in parentheses are the values computed by assuming the position of the optical centre of unit groups as follows: O-atom (in OH group), N-atom (in NH₂ group), Cl-atom, C-atom (in CH₃ group).

TABLE 5. $[\mu]_D^{20} \text{ calcd}(\text{XY})\{3/(n_1^2 + 2)\}$, (= (X) × (Y)), CAUSED BY THE DYNAMIC COUPLING EFFECT BETWEEN TWO UNIT GROUPS, X AND Y, IN CYCLOPENTANE DERIVATIVE

		$Y^{2\beta}$	$Y^{2\alpha}$	$Y^{2\beta}$	$Y^{2\alpha}$
$X^{1\alpha}$		Q	0	P	0
$X^{1\beta}$		0	$-Q$	0	$-P$
X	Y	P^{27}		Q^{27}	
OH	OH	8.35 (4.43)		1.68 (1.09)	
OH	NH ₂	4.09 (2.12)		0.82 (0.52)	
OH	Cl	13.58 (7.69)		3.04 (2.00)	
OH	CH ₃	18.03 (9.02)		3.63 (2.26)	
CH ₃	CH ₃	38.90 (18.42)		7.87 (4.69)	

TABLE 6. $[\mu]_D^{20} \text{ calcd}(\text{XY})\{3/(n_1^2 + 2)\}$, (= (X) × (Y)), CAUSED BY THE DYNAMIC COUPLING EFFECT BETWEEN ANY TWO MEMBERS OF UNIT GROUPS IN (+)-NEOMENTHYLAMINE²²

	(CH ₃) ^{αβ}	(CH ₃) ^{αβ}	(CH ₃) ^{αβ}	(CH ₃) ^{αα}	(NH ₂) ^{βα}	(CH ₃) ^{ββ}
ch.R	0	$-D$	D	0	0	0
(CH ₃) ^{1β}	0	4.41	-4.41	0	-2.00	
(NH ₂) ^{2α}	0	-2.00	1.55	12.39		
(CH ₃) ^{4α}	0	0	0			
(CH ₃) ^{αβ}	0	0				
(CH ₃) ^{αβ}	0		0			
(CH ₃) ^{αβ}		0	0			

By using Table 4, the value of $\kappa_{\text{OH,OH}}$ in the cyclohexane derivative is computed as follows: $[M]_D^{20}$ of (-)-*trans*-1,2-cyclohexanediol, $-54.0^\circ \equiv \sum_{X,Y} [\mu]_D^{20} \text{ obs}(\text{XY})$ of (-)-*trans*-1,2-cyclohexanediol = (OH)^{1β}Λ(OH)^{2α} = (OH)^{1β} × (OH)^{2α}κ_{OH,OH}{(n₁² + 2)/3} = $-11.73 \kappa_{\text{OH,OH}}\{3/(n_1^2 + 2)\}$

$$\therefore \kappa_{\text{OH,OH}} = 4.6036\{3/(n_1^2 + 2)\}. \quad (9)$$

Similarly, by using Tables 4, 5 and 6, the values of all κ's are computed and given in Table 7.

It is apparent from Table 7, that the value of κ_{XY} of Eq. 6 is independent of the compound and is almost constant in a pair of definite groups, X and Y. Thus, it can be said that, in addition to the bond length, the dipole moment of a bond, etc., the value of κ_{XY} is useful in the determination of organic structure (i.e. orientation or conformation).

TABLE 7. $\kappa_{XY}\{3/(n^2 + 2)/3\}$

κ_{XY}	Cyclohexane derivative	Cyclopentane derivative	Pyranose	Menthol-like compound
$\kappa_{OH,OH}$	4.6036	4.1078 _w	4.5183 ²⁸	4.2796 ²³
κ_{OH,NH_2}	8.0522	9.4377		
$\kappa_{OH,Cl}$	ca. 3.0743			
κ_{OH,CH_3}	1.7352	1.7906	1.5440 ²⁹	1.5066 ²³
$\kappa_{NH_2,Cl}$	6.8953			
κ_{NH_2,CH_3}	2.3487 _{al}			2.3642 ²⁰
κ_{CH_3,CH_3}	-0.1552	-0.2922		

w: water solution, al: alcoholic solution.

Next, if Eq. 7 is correct, the values in the first column of Table 7 can be related to the ζ -coefficients as follows:

$$4.6036\{3/(n_1^2 + 2)\} = \zeta_{OH}^2 \quad (10)$$

$$8.0522\{3/(n_1^2 + 2)\} = \zeta_{OH}\zeta_{NH_2} \quad (11)$$

$$\text{ca. } 3.0743\{3/(n_1^2 + 2)\} = \zeta_{OH}\zeta_{Cl} \quad (12)$$

$$1.7352\{3/(n_1^2 + 2)\} = \zeta_{OH}\zeta_{CH_3} \quad (13)$$

$$6.8953\{3/(n_1^2 + 2)\} = \zeta_{NH_2}\zeta_{Cl} \quad (14)$$

$$2.3487\{3/(n_1^2 + 2)\} = \zeta_{NH_2}\zeta_{CH_3} \quad (15)$$

$$-0.1552\{3/(n_1^2 + 2)\} = \zeta_{CH_3}^2 \quad (16)$$

From Eq. 10,
$$\zeta_{OH} = 2.1456\{3/(n_1^2 + 2)\}^{1/2} \quad (10')$$

By substituting Eq. 10' in Eqs. 11, 12 and 13, respectively,

$$\zeta_{NH_2} = 3.7529\{3/(n_1^2 + 2)\}^{1/2} \quad (11')$$

$$\zeta_{Cl} = \text{ca. } 1.4328\{3/(n_1^2 + 2)\}^{1/2} \quad (12')$$

$$\zeta_{CH_3} = 0.8087\{3/(n_1^2 + 2)\}^{1/2} \quad (13')$$

From Eqs. 11' and 12',

$$\zeta_{NH_2}\zeta_{Cl} = \text{ca. } 5.3772\{3/(n_1^2 + 2)\}.$$

This value is of the same order in Eq. 14, $6.8953\{3/(n_1^2 + 2)\}$. Moreover, from Eqs. 11' and 13', the value of $\zeta_{NH_2}\zeta_{CH_3}$ is computed as $3.0350\{3/(n_1^2 + 2)\}$ which is of the

²⁸ S. Yamana, *Bull. Chem. Soc. Japan* **35**, 1421 (1962).

²⁹ S. Yamana, *Bull. Chem. Soc. Japan* **35**, 1269 (1962).

²⁰ By using Tables 3 and 6 and Ref. 20, $[M]_D^{20}$ of (+)-neomenthylamine, $23.5 \equiv \sum_{X,Y} [\mu]_{D^{20}(\text{obs})}^{X,Y}$ of (+)-neomenthylamine $\approx (\text{CH}_3)^{1\beta}\lambda(\text{NH}_2)^{2\alpha} + (\text{CH}_3)^{1\beta}\lambda(\text{CH}_3)^{4\alpha} + (\text{CH}_3)^{1\beta}\lambda(\text{CH}_3)^{\alpha\beta.d} \dots + (\text{CH}_3)^{1\beta}\lambda(\text{CH}_3)^{\alpha\beta.e} + (\text{CH}_3)^{1\beta}\lambda\text{ch.R} + (\text{NH}_2)^{2\alpha}\lambda(\text{CH}_3)^{4\alpha} + (\text{NH}_2)^{2\alpha}\lambda(\text{CH}_3)^{\alpha\beta.d} + (\text{NH}_2)^{2\alpha}\lambda(\text{CH}_3)^{\alpha\beta.e} + (\text{NH}_2)^{2\alpha}\lambda\text{ch.R} + (\text{CH}_3)^{4\alpha}\lambda(\text{CH}_3)^{\alpha\beta.d} + (\text{CH}_3)^{4\alpha}\lambda(\text{CH}_3)^{\alpha\beta.e} + (\text{CH}_3)^{4\alpha}\lambda\text{ch.R} + (\text{CH}_3)^{\alpha\beta.d}\lambda(\text{CH}_3)^{\alpha\beta.e} + (\text{CH}_3)^{\alpha\beta.d}\lambda\text{ch.R} + (\text{CH}_3)^{\alpha\beta.e}\lambda\text{ch.R} = -2.00\{(n_1^2 + 2)/3\}\kappa_{NH_2,CH_3} + 0 - 4.41\{(n_1^2 + 2)/3\}\kappa_{CH_3,CH_3} + 4.41\{(n_1^2 + 2)/3\}\kappa_{CH_3,CH_3} + 0 + 12.39\{(n_1^2 + 2)/3\}\kappa_{NH_2,CH_3} + 1.55\{(n_1^2 + 2)/3\}\kappa_{NH_2,CH_3} - 2.00\{(n_1^2 + 2)/3\}\kappa_{NH_2,CH_3} + 0 + 0 + 0 + 0 + 0 + D - D = 9.94\{(n_1^2 + 2)/3\}\kappa_{NH_2,CH_3} \therefore \kappa_{NH_2,CH_3} \approx 2.3642\{3/(n_1^2 + 2)\}$

same order as that in Eq. 15, $2.3487\{3/(n_1^2 + 2)\}$. These facts indicate that Eq. 7 may be applied and accordingly should be rewritten as

$$\kappa_{XY} = \zeta_X \zeta_Y. \quad (7')$$

The values of ζ_i may, of course, vary according to the type of compounds containing the unit group i . One explanation for the value of κ_{XY} being nearly but not exactly equal to the value of $\zeta_X \zeta_Y$ may be due to the incorrect assumption that all the compounds exist in only the CI conformation. At any rate, the rightness of Eqs. 6 and 7' indicate the availability of the PM-method. On the other hand, from Eq. 13',

$$\zeta_{\text{CH}_3}^2 = [0.8087\{3/(n_1^2 + 2)\}^{1/2}]^2 = 0.6540\{3/(n_1^2 + 2)\}. \quad (17)$$

This value differs from that given in Eq. 16, $-0.1552\{3/(n_1^2 + 2)\}$, and therefore:

$$\kappa_{\text{CH}_3, \text{CH}_3} \neq \zeta_{\text{CH}_3}^2. \quad (18)$$

It is, therefore, apparent that the PM-method is not generally applicable.

The case where the PM-method does not apply

(+)-*cis*-3-methylcyclohexanol has no plane or centre of symmetry and accordingly it is optically active. ($[M]_D = 2.2^\circ$ ³¹). Both of its $\sum_{X,Y} [\mu]_D^{20 \text{ calcd}(XY)}$ and $\sum_{X,Y} [\mu]_D^{20 \text{ obs}(XY)}$ are, however, computed as zero. This discrepancy between the observed and the computed values may be due to the neglect of $\{g^{(1)} + \sum_i g^{(i)}\}$ in Eq. 2. But the case of Eq. 18 is somewhat different.

Being apparent in Table 1, $\sum_{X,Y} [\mu]_D^{20 \text{ obs}(XY)}$ of (+)-*trans*-1,3-dimethylcyclohexane, $I = (\text{CH}_3)^{1\beta} \wedge (\text{CH}_3)^{3\alpha} = (\text{CH}_3)^{1\beta} \times (\text{CH}_3)^{3\alpha} \{3/(n_1^2 + 2)\} = -9.02\{3/(n_1^2 + 2)\} \zeta_{\text{CH}_3}^2$ ³² (19)

When using Eq. 17, the value of

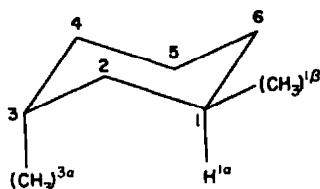
$$\sum_{X,Y} [\mu]_D^{20 \text{ obs}(XY)}$$

of (+)-*trans*-1,3-dimethylcyclohexane in Eq. 19 becomes -5.9 which is quite different from the observed value of $[M]_D^{20}$, 1.4 ³¹. This discrepancy between the computed and the observed values is too large to be explained by the neglect of

$$\{g^{(1)} + \sum_i g^{(i)}\}$$

in Eq. 2, and accordingly the following explanation should be considered.

The CH_3 group has no hydrogen bonding force and moreover its volume is fairly large. In (+)-*trans*-1,3-dimethylcyclohexane, therefore, each one of the two CH_3



I

³¹ cf. Table 1.

³² Table 4 is used.

groups repulses the other H atoms in the molecule. Particularly, the axial CH_3 group (i.e. $(\text{CH}_3)^{3\alpha}$) repulses the $\text{H}^{1\alpha}$ -atom fairly strongly which is under the buttressing effect of the $(\text{CH}_3)^{1\beta}$ and this repulsion may cause some degree of distortion of the ring.³³ In this case, the cyclane ring is not symmetrical as regards to one or both of the two CH_3 groups and the partial rotation, $[\mu]_{\text{D}}^{20}$ due to the dynamic coupling effect between the distorted ring and CH_3 group may have a plus value. This can be the principal reason for the reversion of the sign of optical rotation as is also the case with (+)-*trans*-1,3-dimethylcyclopentane.

³³ R. Bentley, *J. Amer. Chem. Soc.* **82**, 2811 (1960).